

ADVANCED ENGINEERING NUMERICAL METHODS and COMPUTER PROGRAMMING

THEORY AND PRACTICAL



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MACMILLAN

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